Prediction of system reliability for single component repair

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Abstract

Purpose – The purpose of this article is to present a new split system model (SSM) that predicts the reliability of complex systems with multiple preventive maintenance (PM) actions in the long term.

Design/methodology/approach – The SSM was developed using probability theory based on the concept of separating repaired and unrepaired components within a system virtually when modelling the reliability of the system after repairs. After theoretical analysis, a case study and Monte Carlo simulation were used to evaluate the effectiveness of the newly developed model.

Findings – The model can be used to determine the remaining life of systems, to show the changes in reliability with PM actions, and to quantify PM intervals after imperfect repairs.

Practical implications – SSM can be used to predict the reliability of complex systems with multiple PM actions, and hence can be used to support asset PM decision making over the whole life of the asset, such as scheduled PM times and spare parts requirements. An asset often has some vulnerable components, i.e. where the lives of these components are much shorter than the rest of the asset. In this case, PM is often conducted on these vulnerable components for maximising the useful life of the asset. The specific formulae derived in this paper can be used to predict the reliability of the asset for this scenario.

Originality/value – The proposed model uses a new concept of split systems to predict the changes of reliability of complex systems with multiple PM actions. Asset managers will find this model to be a useful tool in the optimisation of their asset PM strategies.

Keywords Reliability management, Preventive maintenance

Paper type Research paper

1. Introduction

In modern asset management, accurate predictability of the reliability of complex repairable systems is desirable because most physical systems in industries are repairable. A repairable system is usually defined as one that can be repaired to recover its functions after each failure rather than be discarded (Crow, 1974). In this paper, "failure" means that the component fails to meet its performance requirement. This "failure" will naturally lead to a need for maintenance. When "repair" is mentioned, it usually includes "replacement". It is important to predict the reliability of complex repairable systems accurately, especially during long periods of operation. A company can plan its production with optimal level of maintenance staffing, inventory and budget according to the prediction of remaining useful life. With increasing complexity of machines and competitive industrial pressure, the need to understand changes in the reliability of a complex repairable system after repairs becomes pressing.

Currently, the most common techniques used to model reliability prediction of repairable systems are based on stochastic or statistical analysis, including renewal

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process models, Markov chain (process) (Bloch-Mercier, 2001, 2002; Bruns, 2002; Juneja and Schahabuddin, 2001; Marquez and Heguedas, 2002), Poisson point process (Hoyland and Rausand, 1994; Van Noortwijk *et al.*, 1995; Saldanha *et al.*, 2001; Weckman *et al.*, 2001), Bayesian method (Van Noortwijk *et al.*, 1995; Percy, 2002; Percy and Kobbacy, 2000; Rosqvist, 2000; Sheu *et al.*, 2001), proportional hazard model (PHM) (Cox and Oakes, 1984; Jardine, 1973; Jardine *et al.*, 1997), and combinations of these models (Guo and Love, 1992; Landers *et al.*, 2001). These different models address reliability prediction of a repairable system using different approaches and have been applied in different scenarios. However, the following two major deficiencies have affected the effectiveness of these existing models. The first deficiency is that the different states of repairable systems after multiple repairs have not been modelled comprehensively. A common approach is to assume that a repairable system after repairs becomes "as good as new" (Bloch-Mercier, 2002). The second deficiency is that existing models often treat a repairable system as a "black box", without considering the individual contributions of different components to the reliability of this system. These two deficiencies will be further analysed in section 2.

The characteristics of the reliability of a system often changes after repairs, rendering difficulties in the prediction of the reliability of complex repairable systems, especially when the prediction covers a number of failures and repairs during an asset's life time. In this paper, a split system model (SSM) is developed to address this issue and to attempt to overcome the two deficiencies discussed earlier.

The rest part of this paper is organised as follows. In section 2, the two deficiencies in the existing models are identified and analysed through an extensive literature review and a case study. Section 3 consists of four subsections. In this section, the concepts of SSM and the assumptions for SSM are introduced. The formula for the reliability prediction of repairable systems under the condition that always the same single component is repaired in all PM actions are derived and two basic applications aspects of SSM are presented. In section 4, the developed model is evaluated using a case study and this is followed by a simulation experiment in section 5. Conclusions are given in section 6.

2. Model deficiencies

2.1 Assumptions for systems after repairs

Existing reliability prediction models often assume that a repairable system after repairs becomes "as good as new" (Armstrong, 2002; Bloch-Mercier, 2002; Weckman *et al.*, 2001), or a similar assumption such that a system after repairs evolves in time according to the same Markovian process as from the beginning (Bloch-Mercier, 2001, 2002). Another common assumption used in existing models is that a system after repairs is "as bad as old" (Hoyland and Rausand, 1994). These assumptions are unrealistic in a considerable number of cases. Often a system after a repair is not as good as new, neither as bad as old leading to the concept of imperfect repair. To date, effective modelling techniques dealing with the reliability prediction of a system with multiple imperfect repairs are still unavailable (Guo and Love, 1992) although some researchers have noticed the influence of imperfect repairs on the reliability of a system (Cox and Oakes, 1984; Kobayashi, 2002; Marquez and Heguedas, 2002; Wang, 2002). Most of existing models are only applied to predict and optimise the next repair activity (Makis and Jardine, 1992; Stavropoulos and Fassois, 2000). The applications of these models are limited and their accuracy of prediction is doubtful if the effects of repairs are not considered. For example, existing non-homogeneous poisson process (NHPP) based models (Cox and Oakes, 1984; Crow, 1974) assume that the number of failures does not affect failure probability and repairs do not change the reliability of a system (Guo and Love, 1992). These models are only suitable for "minimum repair" activities but not general repairs.

Some models consider the influence of imperfect repairs on the reliability of a system, but are not very applicable due to the assumptions used to develop these models. For example, to describe deterioration of reliability of repairable systems after repairs, Artana and Ishida (2002) applied a decreasing percentage (<1) to the original reliability index. Monga *et al.* (1997) assumed the reliability of a system decreased proportionally with repair times and introduced a scale parameter called failure rate deterioration factor. Later, Monga and Zuo (2001) introduced another time variable parameter to describe the different start points of the hazard function of a system after different repairs. Guo and Love (1992) introduced a scalar parameter to reflect the improvement state of a system after repairs similar to Mona's approach. Their model was based on the non-homogeneous Poissonian framework with a proportional intensities assumption. This model regarded the form and parameters of the intensity function of a repairable system as unalterable. In these models, all parameters or factors employed to describe the changes of the reliability function of a system after repairs were normally estimated by maintenance engineers (or users). For complicated systems, estimation of these parameters or factors is difficult, if not impossible, even for experienced personnel. In addition, the assumption that reliability of a system will deteriorate after repairs is not always true. Sometimes the reliability of a repairable system after repair can be better than its original reliability. See subsection 3.2.

2.2 The "black box" approach

A repairable system is often treated as a "black box" in existing models. These models often take the entire system into account and do not analyse the reliability of repairable systems at the component level. As a result, these models lose some important information. The following Nelson-Aalen plot can be used to illustrate this argument. The Nelson-Aalen plot shows the changes of the number of failures of a system with its operational time (see Figure 1). The data presented in Figure 1 are the failure times of a

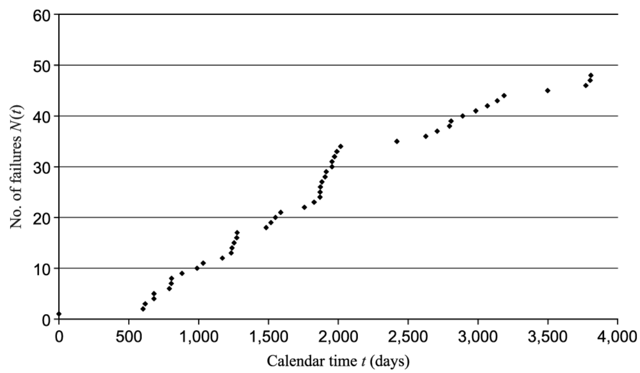

Figure 1.
Number of failures *N(t)* of a pump system versus its age

pump system over nearly ten years. From this figure, it can be seen that the rate of occurrence of failures (ROCOF) of the pump system can be approximated as a constant. However, the reliability changes of the system between two failures cannot be identified from the figure.

The determination of a suitable model to analyse these data is difficult if the pump system is treated as a "black box", because some failure properties can be identified only at the component level. For example, the historical maintenance record of the pump system showed that the fifth failure and the seventh failure in Figure 1 have the same failure modes. Further analysis indicated that these two failures were related because they shared the same root cause. In this case, the assumption of independent, identical distribution (IID) adopted by the homogeneous Poisson process (HPP) model is not valid. In addition, many of the repairs for these failures were not minimal repairs and this indicates that the NHPP model is not suitable.

3. Split system model (SSM)

3.1 Concepts of SSM and assumptions

The above analysis demonstrated that failure and repair information at the component level can often assist in the understanding of the properties of the failure of systems and hence improve the maintenance outcome of these systems. The information at the component level should be considered when the reliability of a system is being modelled. The basic concept of SSM is to separate repaired and unrepaired components within a system virtually when modelling the reliability of the system after repairs. This procedure assists in analysing reliability at the component instead of the system level and comes from the fact that generally when a complex system fails, only some of components fail and need repair. In the following analysis, the following assumptions are made:

- The failures of the repaired components are independent of unrepaired components. This assumption means that when a component is repaired, the failure distribution of the unrepaired part of a system (subsystem) does not change, and the conditions of the subsystem do not affect the reliability characteristics of repaired components.
- The reliability function of a new repairable system is known. The reliability functions of components after repairs are also known.
- The topology of a repairable system is known.

According to the above assumptions, when a system is repaired, only the reliability of the repaired component changes. The reliability of the remainder of the system just before and after this repair does not change.

In addition, the repair time is ignored in the following analysis because this paper focuses on the reliability prediction of a system during its operational period and does not investigate the changes of the reliability of a system during repairs. However, the effects of repair outcomes on reliability will be considered in this study.

This paper focuses on the reliability prediction of a complex system with multiple PM actions over multiple PM intervals. The lead time for these PM actions is a deterministic variable. In this paper, the SSM is developed based on a simple scenario where always the same single component is repaired in all PM actions. The model based on this scenario can be applied to industrial situations, although in this study, the scenario is used to demonstrate the basic concepts and procedures for SSM. An asset often has some vulnerable components, i.e. where the lives of these components are much shorter than the rest of the asset. In this case, PM is often conducted on these vulnerable components for maximising the useful life of the asset. Some practical examples include the preventive maintenance of bulbs in lighting systems, spark plugs in engines, and pads in automotive braking systems that are considered to be weak links in these systems – the same single component is repaired. When modelling the reliability of such a system, the original system can be divided into two parts virtually. "Part 1" is the repaired component and "Part sub" is the remainder of the system, which is often referred to as the subsystem. Both series and parallel systems are considered in this paper.

3.2 Series system

A series system is shown in Figure 2, where $R_1(\tau)_{ai}$, $R_{sub}(\tau)_{ai}$ and $R_s(\tau)_{ai}$ are the reliability functions of repaired component 1 (part 1), subsystem (part sub) and system after the ith repair respectively. In this paper, the subscript ai is used to stand for "after the ith repair". Subscript $a0$ stands for no repair. Parameter τ is a relative time scale (refer to Figure 3).

The situation of imperfect repairs is described in Figure 3. Two time coordinates are used in the modelling:

(1) Absolute time scale t: from 0 to infinite.

(2) Relative time scale τ: from 0 to Δt_i ($i = 1, 2, \ldots, n$).

In Figure 3, R_0 is the required minimum reliability for the system. Parameter Δt_i ($i = 1, 2, \ldots, n$) is the interval between two PM actions. Parameter t_i is the ith PM time and also the start time for a system to run again after the ith repair. Therefore:

$$t = \sum_{i=1}^{n} \Delta t_i + \tau \qquad (1)$$

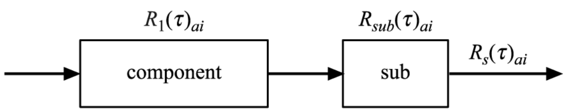

Figure 2.
Series system

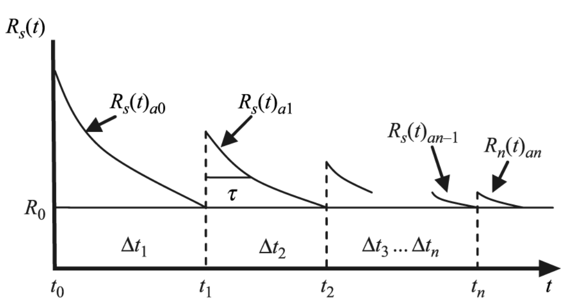

Figure 3.
Changes to the reliability of an imperfectly repaired system

Initially, the reliability function of a system can be expressed as:

$$R_s(\tau)_{a0} = R_1(\tau)_{a0} R_{sub}(\tau)_{a0} \tag{2}$$

At time t_1, the reliability of the system falls to the required minimum level R_0 of reliability, and component 1 is repaired so that:

$$R_1(0)_{a1} > R_1(t_1)_{a0} \tag{3}$$

Hence, after the first repair, the reliability of the system becomes:

$$R_s(\tau)_{a1} = \frac{R_1(\tau)_{a1} R_s(\tau + t_1)_{a0}}{R_1(\tau + t_1)_{a0}} \tag{4}$$

The following equation holds because reliability decreases monotonously with the increase of operational time:

$$R_{sub}(\tau + t_1)_{a0} = \frac{R_s(\tau + t_1)_{a0}}{R_1(\tau + t_1)_{a0}} < R_{sub}(\tau)_{a0} = \frac{R_s(\tau)_{a0}}{R_1(\tau)_{a0}} \tag{5}$$

$$\text{If } R_1(\tau)_{a1} = R_1(\tau + t_1)_{a0}, \text{then } R_s(\tau)_{a1} = R_s(\tau + t_1)_{a0}$$

This indicates that the system is repaired as bad as old.

If component 1 is repaired or replaced by an identical one:

$$R_1(\tau + \Delta t_1)_{a0} < R_1(\tau)_{a1} \leq R_1(\tau)_{a0}$$

In this case, equation (4) represents the situation that systems are repaired imperfectly because in this case:

$$R_s(\tau + \Delta t_1)_{a0} < R_s(\tau)_{a1} < R_s(\tau)_{a0}$$

If the reliability of component 1 after the repair $R_1(\tau)_{a1}$ is better than its initial reliability $R_1(\tau)_{a0}$, so that:

$$R_s(\tau)_{a1} \geq R_s(\tau)_{a0}$$

Equation (4) then represents the case where the state of a system after repairs is improved to be as good as new, or even better than original new one. As a result, equation (4) can describe different possible states of a system after repairs.

The reliability function of system after the n[th] repair can be derived as:

$$R_s(\tau)_{an} = \frac{R_1(\tau)_{an} R_s(\tau + \sum_{i=1}^{n} \Delta t_i)_{a0}}{R_1(\tau + \sum_{i=1}^{n} \Delta t_i)_{a0}} \tag{6}$$

Equation (6) can be rewritten in the following form using the absolute time scale:

$$R_s(t) = \frac{R_1(t - \sum_{i=1}^{n} \Delta t_i)_{an} R_s(t)_{a0}}{R_1(t)_{a0}}, \left(t > \sum_{i=1}^{n} \Delta t_i\right) \tag{7}$$

The function $R_s(t)$ indicates the reliability of a system after the n[th] PM interval.

Obviously, component 1 should not be repaired any more if the reliability of the system after this repair cannot be recovered to above the required minimum reliability level, i.e.:

$$R_s(0)_{an} \leq R_0$$

or

$$\frac{R_1(0)_{an} R_s(\sum_{i=1}^{n} \Delta t_i)_{a0}}{R_1(\sum_{i=1}^{n} \Delta t_i)_{a0}} \leq R_0 \tag{8}$$

3.3 Parallel system

The schema of a parallel system is shown in Figure 4.

The relationship among reliability functions $R_1(\tau)_{ai}$, $R_{sub}(\tau)_{ai}$ and $R_s(\tau)_{ai}$ is given by:

$$R_s(\tau)_{ai} = R_1(\tau)_{ai} + R_{sub}(\tau)_{ai} - R_1(\tau)_{ai} R_{sub}(\tau)_{ai}, \ (i = 1, 2, \ldots, n) \tag{9}$$

To simplify mathematical operations, let $F_1(\tau)_{ai}$, $F_{sub}(\tau)_{ai}$ and $F_s(\tau)_{ai}$ be corresponding failure distribution functions of component 1, subsystem and system respectively. According to reliability theory, equation (9) becomes:

$$F_s(\tau)_{ai} = F_1(\tau)_{ai} F_{sub}(\tau)_{ai}, \ (i = 1, 2, \ldots, n) \tag{10}$$

Based on the same derivation procedure as in subsection 3.2, the following results can be obtained (see Figure 5):

$$F_s(\tau)_{a1} = \frac{F_1(\tau)_{a1} F_s(\tau + t_1)_{a0}}{F_1(\tau + t_1)_{a0}} \tag{11}$$

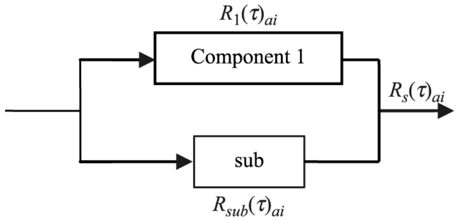

Figure 4.
Parallel system

$$F_s(\tau)_{an} = \frac{F_1(\tau)_{an} F_s(\tau + \sum_{i=1}^{n} \Delta t_i)_{a0}}{F_1(\tau + \sum_{i=1}^{n} \Delta t_i)_{a0}} \tag{12}$$

$$F_s(t) = \frac{F_1(t - \sum_{i=1}^{n} \Delta t_i)_{an} F_s(t)_{a0}}{F_1(t)_{a0}}, \left(t > \sum_{i=1}^{n} \Delta t_i\right) \tag{13}$$

where, F_0 is the control level of the accumulative failure probability of a system. Functions $F_s(\tau)_{a0}$, $F_s(\tau)_{a1}$, and $F_s(\tau)_{an}$ are the failure distribution functions of the original system, the system after the first repair, and after the n^{th} repair respectively. Function $F_s(t)$ is the failure distribution function of the system with multiple PM intervals, and presented in the absolute time scale. Functions $F_1(\tau)_{a0}$, $F_1(\tau)_{a1}$ and $F_1(\tau)_{an}$ represent the failure distribution functions of the original component 1, and component 1 after the first repair and after the n^{th} repair, respectively.

Equation (13) can be rewritten in the term of reliability function as follows:

$$R_s(t) = 1 - \frac{(1 - R_1(t - \sum_{i=1}^{n} \Delta t_i)_{an})(1 - R_s(t)_{a0})}{1 - R_1(t)_{a0}}, \left(t > \sum_{i=1}^{n} \Delta t_i\right) \tag{14}$$

Generally, $F_1(0)_{ai} < F_1(t_1)_{ai-1}$ and the failure distribution function of the subsystem, $F_{sub}(\tau)$ increases monotonously with the increase of operational time, so:

$$F_s(t_i)_{ai-1} > F_s(0)_{ai} > F_s(0)_{ai-1}, \ (i = 1, 2, \ldots, n) \tag{15}$$

Equation (15) indicates that a system is repaired imperfectly. It is noted that equations (12), (13) or (14) can also represent different states of a system after repairs due to the similar reasons mentioned in subsection 3.2.

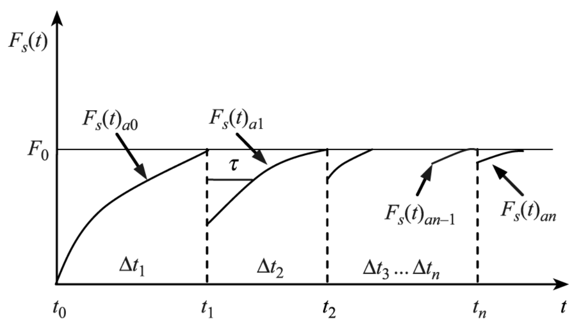

Figure 5.
Changes of the failure distribution function of an imperfectly repaired system

3.4 SSM applications

The model developed in this study can be used for the following applications:

- To improve the accuracy of estimating reliability of a repairable system using historical failure data. Historical failure data (failure times) of a system can be used to estimate the reliability of this system using statistical methods if these failures are IID and the system becomes as good as new after each repair. Life data from industry hardly meet this constraint because of imperfect repairs and related failures as presented in Figure 1. If these industrial failure data are directly used to estimate the failure distribution function of a system, the reliability estimation will most likely be in error. SSM can remove these constraints and this is best illustrated by an example. A system has an imperfect repair after failure time $t_{n\text{-}1}$ and then fails at time t_n. On average, the failure interval $\Delta t_n = t_n - t_{n-1}$ is assumed to be shorter than if this system is repaired as good as new (the mean time between two failures when this system is assumed to be repaired as good as new is termed as "should-be-failure-interval"). The difference between "should-be-failure-interval" and real failure interval can be calculated using SSM since the reliability function of the system between time $t_{n\text{-}1}$ and t_n can be estimated using equation (9). Therefore the "should-be-failure-time" and its variation can be found and used for statistical analysis. More sophisticated techniques of the application of SSM to improve the accuracy when estimating the reliability of a repairable system using historical failure data are currently being developed by the authors.
- To support preventive maintenance decision making for a repairable system during its lifetime. While the most of prediction models focuses on the next failure and/or next PM action only, the multiple PM lead times of a repairable system over its life span can be predicted using SSM. Therefore, the optimal level of maintenance staffing, inventory and budgets can be planned in advance. The capability of SSM to support preventive maintenance decision making for a repairable system over its lifetime is demonstrated by the following case study in section 4.

4. A case study

A reparable mechanical system is the same as described in subsection 3.2. Preventive maintenance is used to reduce the failure probability of the system. Whenever the reliability of the system falls to the required minimum reliability level R_0, a PM action is conducted. When a PM action is taken, component 1 is always replaced by an identical new one. The reliability functions of the original system and component 1 are:

$$R_s(\tau)_{a0} = \exp[-\lambda_s \tau - (\lambda_1 \tau)^2] \quad (16)$$

and

$$R_1(\tau)_{a0} = \exp[-(\lambda_1 \tau)^2] \quad (17)$$

The original reliability function of subsystem can be found according to reliability theory:

$$R_{sub}(\tau)_{a0} = \exp(-\lambda_s \tau) \quad (18)$$

The first PM lead time $t_1 = \Delta t_1$ is given by:

$$t_1 = \Delta t_1 = \frac{-\lambda_s + \sqrt{\lambda_s^2 - 4\lambda_1^2 \ln R_0}}{2\lambda_1^2}, (1 > R_0 > 0) \tag{19}$$

Using equation (6), one has:

$$\begin{aligned} R_s(\tau)_{a1} &= \frac{\exp[-(\lambda_1\tau)^2]\exp[-\lambda_s(\tau+\Delta t_1) - \lambda_1^2(\tau+\Delta t_1)^2]}{\exp[-\lambda_1^2(\tau+\Delta t_1)^2]} \\ &= R_s(\tau)_{a0}\exp(-\lambda_s\Delta t_1) \end{aligned} \tag{20}$$

When the system is at its first PM action, $R_s(\tau = t_1)_{a0} = R_0$. The reliability of the system just after this PM action is:

$$R_s(0)_{a1} = \exp(-\lambda_s\Delta t_1) = R_0 \exp(\lambda_1\Delta t_1)^2 > R_0 \tag{21}$$

The reliability of the system after repair increases but is not restored to 1 (the initial reliability level of the system) because $1 > \exp(-\lambda_s\Delta t_1) > 0$.

The interval between the first PM action and the second PM action is found by using equation (20):

$$\Delta t_2 = \frac{-\lambda_s + \sqrt{\lambda_s^2 - 4\lambda_1^2(\ln R_0 + \lambda_s\Delta t_1)}}{2\lambda_1^2} \tag{22}$$

Obviously, $\Delta t_2 < \Delta t_1$ due to Δt_1 and λ_s are both greater than zero, that is, the interval time between two PM actions becomes shorter after repairs if the same minimum reliability level needs to be maintained. The reliability function of the repairable system after the n$^{\text{th}}$ PM action is given by:

$$R_s(\tau)_{an} = R_s(\tau)_{a0}\exp(-\lambda_s\sum_{i=1}^{n-1}\Delta t_i) \tag{23}$$

If the absolute time scale is applied, equation (23) can be rewritten as:

$$R_s(t) = R_s(t)_{a0}\exp\left[2\lambda_1^2 t\sum_{i=1}^{n}\Delta t_i - \left(\lambda_1\sum_{i=1}^{n}\Delta t_i\right)^2 + \lambda_s\Delta t_n\right], \left(t > \sum_{i=1}^{n}\Delta t_i\right) \tag{24}$$

If component 1 ceases to be produced, how many spare parts of component 1 should be kept for the life span of the system if only PM is considered? The answer can be found using the following criterion. The interval time between two PM actions must be longer than required minimum operational time t_p, that is:

$$\Delta t_n \geq t_p \tag{25}$$

The interval time between two PM actions is:

$$\Delta t_n = \frac{-\lambda_s + \sqrt{\lambda_s^2 - 4\lambda_1^2(\ln R_0 + \lambda_s \sum_{i=1}^{n-1} \Delta t_i)}}{2\lambda_1^2} \tag{26}$$

Therefore, the maximum number N of component 1 to be stored can be estimated by solving the following inequality:

$$\frac{-\lambda_s + \sqrt{\lambda_s^2 - 4\lambda_1^2\left(\ln R_0 + \lambda_s \sum_{i=1}^{n-1} \Delta t_i\right)}}{2\lambda_1^2} \geq t_p \tag{27}$$

Inequality equation (27) needs to be solved recurrently and numerically. An example is given as follows.

For this mechanical system, $\lambda_1 = 0.0008$ (1/day) and $\lambda_s = 0.00011$ (1/day). The required minimum reliability level R_0 is 0.9. Table I shows the relationship between the number of spare parts and the required minimum operational time t_p.

Since the maintenance strategy in this case study is assumed to replace component 1 in all PM actions. The subsystem will keep deteriorating over its operational time. Finally, the deterioration of the subsystem will become so bad that the reliability of the system can no longer be maintained to above the required minimum level R_0 by replacing component 1 only. In this situation, the expected life T of this repairable system can be estimated according to the following formula:

$$T = \sum_{i=1}^{n+1} \Delta t_i \tag{28}$$

If the required minimum operational time is 72 days, the expected life of the system is 880.3 days, which is more than 2.5 times of the age when the reliability of system first falls to the control level of reliability R_0 (328.8 days).

5. Simulation test

The SSM was evaluated using a series of Monte Carlo simulation experiments. Figure 6 presents an example result of such simulation experiments and the corresponding analytical result.

From this figure, it can be concluded that SSM identifies the same number of failures, consistent with the simulation results. The characteristics of the failure distribution of the system predicted by SSM are very close to the results of the

Table I. The relationship between the spare parts N and the required minimum operational time tp

t_p (days)	30	50	72	100	120	150	180
N	5	5	4	4	3	3	3

simulation experiment. The maximum relative error of the reliability estimated using SSM to the simulation results was less than 2.54 per cent.

Another point of comparison is the times when the reliability of the system reaches the minimum required level R_0 after repairs. If that point is t_{SSM} and t_{SIM} estimated by SSM and the simulation experiments respectively, then the maximum absolute relative error of t_{SSM} to t_{SIM} was 6.06 per cent (see Table II), which is acceptable. This result confirms that SSM has a commendable accuracy of prediction.

6. Conclusion

SSM as proposed, can predict the reliability of a repairable system at the component level. Compared with existing models, SSM has the following advantages:

- It is able to explicitly predict the reliability of a repairable system with multiple PM actions over a long term. It can be used to decide when the system is unworthy of further PM from the reliability point of view (refer to equation (8)), whereas most of the existing models are only effective in predicting the next repair activity. However, a repairable system often experiences several failures and PM actions over its operating life.
- It is able to deal with the individual contributions of different parts in a system and the influence of system structures on the reliability of a repairable system. This ability provides an understanding of the impact of repairs on the reliability of a system in more depth compared to the "black box" approach commonly used. As a result, SSM can be used for evaluating the reliability of a system at the component level.
- It is able to model different states of a system after repairs such as "as good as new", "imperfect repair", "improvement repair" (i.e. better than new) and "as bad as old".
- It is not restricted by the forms of failure distribution.

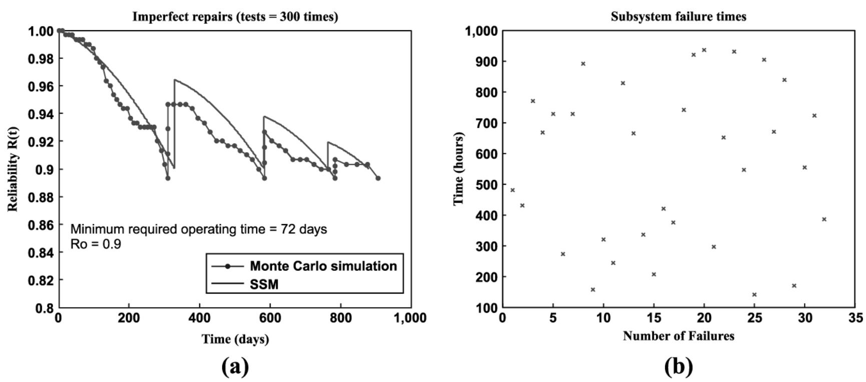

Figure 6. An example of simulation experimental results: (a) the changes of the reliability of a system over its whole life cycle; (b) the failure times of the subsystem

Table II. The times when the reliability of the system dropped to the minimum required reliability level after repairs

t_{SSM} (days)	328.8	582.7	764.8	875.8
t_{SIM} (days)	310	584.2	785.6	905.8
Error (%)	6.06	−0.26	−2.65	−3.31

SSM can improve the accuracy of estimating the reliability function of a repairable system using historical failure data. It can be extended to support preventive maintenance decision making for a repairable system over its whole life. As such, the expected life of a repairable system with multiple PM actions over multiple intervals can be estimated using SSM. Also, the available PM times of a system and the spare parts requirement can be predicted.

This paper reports on work concerning the scenario where a system has been preventively maintained for n times, i.e. the conditional probability of survival of a system with PM actions over multiple PM intervals. The authors have also conducted work on the cumulative effect of failure probability of repaired components over time. These results will be published in due course.

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